Automated proton treatment planning with robust optimization using constrained hierarchical optimization

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Robust optimization

- Intentionally impose errors
- Stochastic and the worst-case approach

**Worst-case:**

\[
\min_x \left( \max_s (f_s(x)) \right)
\]

With \( x \) the proton spot weights, and \( s \) the scenario index

**Stochastic:**

\[
\min_x \left( \mathbb{E}(f_s(x)) \right) = \min_x \left( \sum_s w_s \cdot f_s(x) \right)
\]

with \( \mathbb{E} \) the expectation value, and \( w_s \) an importance weight (or probability) for scenario \( s \)
Robust optimization

Stochastic:
• Includes all scenarios
• Focuses on the average scenario

Worst-case:
• Includes only one scenario
• Focuses on worst scenario
Robust optimization – our approach

Robust optimization:
- Includes the worst-case scenario
- Focuses on the worst outcome

Stochastic:
- Includes all scenarios
- Focuses on the average scenario

Worst-case:
- Includes only one scenario
- Focuses on worst scenario

Robust optimization:
\[
\min_x \left( \sum_{s \in S} (f(d(x, s)))^p \right)^{\frac{1}{p}}
\]
**ECHO** (Expedited Constrained Hierarchical Optimization)

- Constrained optimization
  - Max and mean doses are strictly fulfilled
  - Tuning of objective weights ($b_w$) is avoided

- Hierarchical optimization
  1. Target coverage
  2. OAR sparing

Robustness is included in both steps
Patients

Scenarios:
1 nominal
6 x 3 mm setup error + 3.5% range error
6 x 3 mm setup error + -3.5% range error

Beam directions
Robust vs non-robust

Nominal scenario better for the non-robust optimization

Non-robust vs robust: Patient 2, CTV

Robust

Non-robust

Less spread over the scenarios for the robust optimization

Non-robust vs robust: Patient 2, Parotid L

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Robust vs non-robust

Non-robust vs robust: Water phantom, CTV

Non-robust vs robust: Water phantom, OAR
**p-norm (and stochastic) vs worst-case**

**Stochastic: p = 1**

CTV: Worst-case vs p-norm \((p = 1)\)

CTV: Worst-case vs p-norm \((p = 2)\)

CTV: Worst-case vs p-norm \((p = 10)\)

Parotid R: Worst-case vs p-norm \((p = 1)\)

Parotid R: Worst-case vs p-norm \((p = 2)\)

Parotid R: Worst-case vs p-norm \((p = 10)\)
$p$-norm (and stochastic) vs worst-case

Patient 1

Patient 2

Patient 3

Phantom

Obj. func. = square sum of deviations (area marked in red)

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**p-norm (and stochastic) vs worst-case**

**Prescription dose** = 70 Gy

**Worst-case approach:**
- Less variation among the scenarios
- Better in the worst scenario
- Worse median value

**p-norm approach:**
- Full flexibility to focus on the most important criteria
Summary

♦ ECHO – automated treatment planning for protons
♦ Robustness approach in-between extreme approaches
♦ Flexibility to balance between the nominal and the worst scenario

Thank you very much for your attention
References


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